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THE LOGIT AS A MODEL OF PRODUCT DIFFERENTIATION

By SIMON P. ANDERSON *and* ANDRÉ DE PALMA*

1. Introduction

THIS paper presents a simple approach to describe the demand of a population of heterogeneous consumers for a set of differentiated goods. The demand system used is the multinomial logit model, which is the best known probabilistic discrete choice model (see Manski and McFadden (1981) or McFadden (1980)). The logit has been extensively used in econometric studies to describe the demand of individuals facing discrete choices. Examples include Berkovec (1985) and Train (1986) for applications to the demand for automobiles and Train et al. (1987) for telecommunications demand models. Here we study the *supply* response of firms facing such a demand system: in looking at market equilibrium we can endogenize the prices faced by consumers.

The logit is frequently used by researchers in marketing (see Ben Akiva and Lerman (1986) and McFadden (1986)). Insofar as some of this research is commissioned by firms (and presumably acted upon), their pricing and production decisions will be influenced by this approach. Since we first proposed using the logit as a tool for modelling competition under product differentiation (see de Palma et al. (1985) in location theory, and Anderson and de Palma (1985) for an early version of this paper), we have become convinced that the discrete choice approach should play a key role in industrial organization.

Recent work has clarified the exact nature of the links between the address (or characteristics) approach, the representative consumer approach and the discrete choice approach. Anderson et al. (1989) show that, if the logit model is to be derived from an address approach, then the dimension of the characteristics space over which consumer preferences are defined must be at least $n - 1$, where n is the number of products in the market. In this sense the logit model is very different from the one-dimensional circle model considered for example by Salop (1979). Not surprisingly, this will affect the equilibrium price level and welfare analysis. The connection with the representative consumer model was shown by Anderson et al. (1988). The direct utility function associated with the logit model has an entropic form; it is increasing with the variety of goods available to the representative consumer. Consumer taste for variety is measured by a key parameter denoted by μ. When μ is zero, the representative consumer does not value variety at all and buys a single product. When μ becomes very large, the representative consumer is primarily interested in variety, and will then typically consume an equal amount of all products. All intermediary cases are also described by the logit formulation

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which thus provides a very simple parameterization of diversity in consumer tastes.

These contributions provide a strong justification for using the logit approach as a model of product differentiation, and on these grounds we believe it is a serious contender with existing models. Our study is complementary to the analyses of Perloff and Salop (1985) and Sattinger (1984). In Perloff and Salop, all consumers must purchase from one of the firms. Apart from this limitation, these authors use a more general functional form than ours; however, this leads to a relative paucity of predictions. Moreover, they do not treat any welfare issues. Sattinger does address welfare issues, but his demand system does not allow the derivation of explicit results. However, we feel that explicit results are important. For example we find that the market equilibrium may provide too much or not enough variety. Further analysis shows that when the market provides too much variety, the excess number of firms cannot be larger than one!

In the next section we set out the model and discuss the properties of the logit in an oligopoly setting. In Section 3, the existence of a symmetric equilibrium is proved and comparative static analysis is undertaken. Section 4 provides analysis of free entry equilibrium. Section 5 considers first and second best social optima. In Section 6, the results of the two previous sections are compared and contrasted, with special regard to the question of equilibrium and optimal product diversity. Section 7 presents final discussion and conclusions.

2. The Model

We are concerned with analyzing an oligopolistic industry comprising n distinct firms labelled $i = 1, .., n$, each of which produces a single product differentiated from those offered by the other firms in the sense that consumer tastes are heterogeneous over these commodities. There are N consumers, each of whom purchases a single unit from one of the n firms or else chooses not to buy. Not buying is analogous to there being an 'outside good' in Salop's (1979) terminology. As a special case, the model admits the possibility that all consumers buy. We shall also assume that residual consumer income is spent on a numeraire commodity, and that prices are low enough that all consumers can afford each product.

2.1. *Consumer demand*

First consider a model where all consumers have the same tastes. The utility of a consumer purchasing a product from firm i with price p_i is:

$$V_i = a - p_i, \quad i = 1, \ldots, n, \tag{2.1}$$

where a is assumed identical across consumers and products.[1]

[1] In a more general context, the parameter a could differ across products and consumers. Advertising and brand loyalty may be important determinants of this variable. These extensions are a subject for future research.

The utility level associated with non-purchase is denoted by V_0. Under the hypothesis that all consumers have the same tastes, individuals will choose option i, $i = 0, \ldots, n$, if:

$$V_i > V_j, \quad j = 0, 1, \ldots n, \quad j \neq i. \tag{2.2}$$

Therefore, unless the utility derived from two or more products is the same, the whole demand, N, will be addressed to a single firm.

We now allow for consumers to have idiosyncratic tastes. Indeed, there are many characteristics of products which consumer value differently, such as color, aesthetic appeal, brand image, etc. The manner in which consumers evaluate the large number of different characteristics inherent in any product is not systematic and varies across individuals.

Bearing in mind these considerations, the utility derived from the purchase of a particular product is not the same for all individuals. In view of the bewildering array of factors influencing consumer demand, the best that can be done by the firm—after accounting for the factors which are perceived in the same manner by all consumers—is to model these other influences as stochastic. Hence we assume that an individual's utility from purchasing good i, as modelled by the firm, is given by:

$$U_i = V_i + \mu\varepsilon_i, \quad i = 0, 1, \ldots n, \tag{2.3}$$

where V_i is given by (2.1); ε_i is a random variable of zero mean and unit variance and μ is a positive parameter (indeed this is the econometric approach). Note that non-purchase is also assumed to yield a utility value which the firm does not know with certainty (an alternative assumption—that $U_0 = V_0$—is considered by Besanko, Perry and Spady (1990)).

Because the non-systematic factors are *a priori* very numerous, differing from product to product and from individual to individual (and may even change between two consecutive choices made by the same individual) the stochastic factors, ε_i, will be assumed to be identically independently distributed across choices and individuals.

The term $\mu\varepsilon_i$ expresses horizontal differentiation between products. As μ increases, horizontal differentiation plays a more important role in that consumer tastes are more heterogeneous so consumers are less responsive to the price of the product and more responsive to non-systematic factors. Now, each firm is only able to model the *probability*, $\mathscr{P}_i$, that a given individual will purchase its product.[2]

Following, for example, Manski and McFadden (1981), we assume that the distribution of ε is identically, independently Gumbel distributed. Let $\mathbf{p}$ denote

[2] This probabilistic approach was suggested in location theory by de Palma et al. (1985) and Anderson and de Palma (1988) to describe location and price competition between firms selling differentiated products. A similar framework has been used by Besanko and Perry (1988) to look at exclusive dealing by retailers. In these models the spatial dimension constitutes a second source of horizontal differentiation.

the price vector $(p_1, \ldots, p_n)$. The probability of a consumer choosing option i, $\mathscr{P}_i(\mathbf{p}, V_0)$, $i = 0, 1, \ldots n$, is then given by the multinomial logit model as

$$\mathscr{P}_i(\mathbf{p}, V_0) = \frac{\exp[(a - p_i)/\mu]}{\sum_{j=1}^{n} \exp[(a - p_j)/\mu] + \exp[V_0/\mu]}, \quad i = 1, \ldots n; \tag{2.4}$$

the probability of not purchasing is $\mathscr{P}_0(\mathbf{p}, V_0) = 1 - \sum_{i=1}^{n} \mathscr{P}_i(\mathbf{p}, V_0)$. The case where everyone buys corresponds to the case $V_0 \to -\infty$ (and therefore $\mathscr{P}_0 \to 0$). For simplicity, we shall henceforth omit the arguments of the probability functions.

Before proceeding further, we shall discuss some simple properties of the logit formulation. We first study the effects of varying μ, the degree of heterogeneity in consumer tastes. As $\mu \to 0$, all consumers will choose the option with the greatest value of V_i, $i = 0, 1, \ldots n$. That is, the model reverts to the deterministic model discussed at the beginning of this section. For $\mu \to \infty$, we have from (2.4) that:

$$\lim_{\mu \to \infty} \mathscr{P}_i = \frac{1}{n+1};$$

that is, if heterogeneity in tastes is very large, the deterministic part of the utility function, V_i, has no predictive power and consumers behave as if they were completely random. We now introduce a lemma which is used throughout the paper.

Lemma 1

$$\frac{\partial \mathscr{P}_i}{\partial p_i} = \frac{-\mathscr{P}_i(1 - \mathscr{P}_i)}{\mu}, \quad i = 1, \ldots n \tag{2.5}$$

$$\frac{\partial \mathscr{P}_i}{\partial p_j} = \frac{\mathscr{P}_i \mathscr{P}_j}{\mu}, \quad j \neq i, j \neq 0, i, j = 0, 1 \ldots n. \tag{2.6}$$

The proofs are straightforward.

2.2. *Elasticity analysis*

It was claimed earlier that the functional form analyzed in this paper exhibits properties which are reasonable for an industry competing with differentiated products. We now substantiate this claim by analyzing the properties of the demand elasticities given from the model. We assume, for the purpose of the demonstration, that elasticity is measured at the point where all firms have the same price, p, and output $N\mathscr{P}$.[3] The reason for analyzing this case is made clear in the next section.

[3] In fact the signs of the elasticities given in the following analysis always hold in the model: only the explicit equations will differ.

Consider first the elasticity of the Chamberlinian DD curve, that is, when all prices change by the same amount. Using Lemma 1 we have:

$$\varepsilon_{DD} = \frac{\mathrm{d}N\mathscr{P}}{\mathrm{d}p}\frac{p}{N\mathscr{P}}\bigg|_{\text{sym}} = -p\frac{(1-n\mathscr{P})}{\mu} < 0.$$

Note that without the nonpurchase option ($V_0 \to -\infty$), the *DD* curve is completely inelastic since $\mathscr{P} = 1/n$. The elasticity of the Chamberlinian *dd* curve—when one firm's price varies with all others constant—is given by

$$\varepsilon_{dd}^{ii} = \frac{\mathrm{d}N\mathscr{P}_i}{\mathrm{d}p_i}\frac{p_i}{N\mathscr{P}_i}\bigg|_{\text{sym}} = -p\frac{(1-\mathscr{P})}{\mu} < 0.$$

Comparing the two elasticities shows the *dd* curve is more elastic than the *DD* curve, as expected. The cross elasticity of demand is given by

$$\varepsilon_{dd}^{ij} = \frac{\mathrm{d}N\mathscr{P}_i}{\mathrm{d}p_j}\frac{p_j}{N\mathscr{P}_i}\bigg|_{\text{sym}} = \frac{p\mathscr{P}}{\mu} > 0,$$

which shows that products are symmetric substitutes at a symmetric solution. Note that as $n \to \infty$ then $\mathscr{P} \to 0$ and hence $\varepsilon_{dd}^{ij} \to 0$: the individual cross effects are negligible under conditions of monopolistic competition. This assumption is effectively used by Dixit and Stiglitz (1977) in their analysis.

Returning to the own elasticity of the *dd* curve we have:

$$\frac{\partial \varepsilon_{dd}^{ii}}{\partial n} = \frac{p}{\mu}\frac{\partial \mathscr{P}}{\partial n} < 0.$$

The more firms in the market, the more elastic the demand curve for each individual firm. This accords well with intuition: conditions are getting closer to perfect competition. (This property is not a feature of the Dixit-Stiglitz model—see their equation (9), p. 299, and Pettengill (1979) criticized their analysis on these grounds.) As non-purchase becomes more attractive (an increase in V_0) then demand becomes more elastic. The response of the *dd* curve to changes in μ is a little more complex, as will be seen in the next section.

2.3. *The production technology and profit functions*

All firms are assumed to produce with the same constant marginal cost, c. Setting up a firm entails a fixed cost, K. Noting that $N\mathscr{P}_i$ is the expected demand addressed to a firm,[4] the profit of firm i is given by:

$$\pi_i = [p_i - c]N\mathscr{P}_i - K, \quad i = 1 \ldots n \tag{2.7}$$

where $\mathscr{P}_i$ is given by (2.4). The equilibrium is Nash in prices.

[4] The firms are assumed to be risk neutral in the case where consumer demand is probabilistic. This assumption is not necessary if the model is interpreted as in the introduction.

3. Equilibrium for Fixed Numbers of Firms

In this section we derive the equilibrium and discuss the comparative static properties of the model. We assume for now the number of firms is fixed at n. This can be viewed as a short-run equilibrium, and also helps explain the results of the free entry equilibrium analyzed in the next section.

Proposition 1. There exists a unique equilibrium which is the solution to

$$p^* = c + \frac{\mu}{1 - \mathscr{P}^*} \tag{3.1}$$

where $\mathscr{P}^*$ is given by (2.4) *as*

$$\mathscr{P}^* = [n + \exp[(V_0 - a + p^*)/\mu]]^{-1} \tag{3.2}$$

Proof: see Appendix 1.

Proposition 1 proves that the equilibrium is symmetric. It is worth noting that as $V_0 \to -\infty$, that is, when all customers purchase the differentiated product, the equilibrium prices reduces to:

$$p^* = c + \frac{\mu n}{n - 1}. \tag{3.3}$$

In this case, as heterogeneity of consumer tastes (measured by μ) tends to zero, price tends to marginal cost, as expected in Bertrand competition with homogeneous products. When μ is strictly positive, the equilibrium price does not tend to c, even as $n \to \infty$ (see also (3.1), which shows the same property for V_0 finite). Hence the limit case of the present model is Chamberlinian monopolistic competition rather than perfect competition.

We now explore the comparative static properties of the equilibrium. The comparative static effects on price, output per firm and profits are shown in Table 1. The proofs are straightforward.

Note also that total industry output rises as n increases. The effect on total industry revenues net of variable production costs (TIR) is rather interesting and is described intuitively below. The role of the non-purchase option is critical. If $V_0 \to -\infty$ then $\mathscr{P}^* \to 1/n$, and adding a firm depresses the equilibrium price whilst the total market size remains constant at N. Here TIR decreases

TABLE 1
Comparative statics of short-run equilibrium

	Output per Firm $N\mathscr{P}$	*Price* p	*Profit per Firm* π
N	+	0	+
$V_0 - a$	−	−	−
c	−	+	−
n	−	−	−

with n. On the other hand, when not buying is relatively attractive, a further firm will obtain sales primarily from those consumers previously patronizing the external sector rather than competing firms. Effects on equilibrium price tend to be small so that the total net revenue obtained from the market rises. More generally, for V_0 finite, TIR will first increase and then (eventually) decrease with n.

An increase in the parameter μ shows the effect of increasing the intensity of preferences, or, more literally, increasing the heterogeneity of consumer tastes. When $V_0 \to -\infty$, the comparative static effects are clear from (3.3): all prices rise and so do profits, output per firm remaining constant. This is because increasing μ leads to more inelastic demands (more 'loyal' customers in a sense) and firms respond by increasing prices (this result is also obtained by Perloff and Salop (1985) for the case $V_0 \to -\infty$, but with more general demands otherwise). When V_0 is finite (that is, some consumers do not purchase), the effects of a change in μ are more complex.

First we still have the effect noted above that demand becomes more inelastic. Second though, increasing μ tends to spread demand more evenly across products (including the outside option) for any given set of prices. This means demand for the variants will tend to shift in (out) for $V_0 - a + p^* < 0 (> 0)$. Basically, quality differences $(V_0 - a)$ become less important in determining choice.

For $V_0 - a + p^* < 0$, that is, if non-purchase is relatively unattractive, firm output necessarily decreases as μ rises because demand has shifted in (more customers are drawn to not buy) and becomes more inelastic. The effects on prices and profit are ambiguous *a priori*; however, as shown in Anderson and de Palma (1985), both of these magnitudes necessarily rise as long as there are two or more firms. For a monopolist, price and profit do not necessarily increase with μ when its product is relatively attractive vis-a-vis not buying. In the monopoly case, the firm does not benefit from the reduction of intra-industry competition which occurs in oligopoly with increases in μ. Hence we only have the effect on competition with non-purchase, which is ambiguous, although price and profit necessarily increase with μ for μ large enough.

For $V_0 - a + p^* > 0$, which is the case where not buying is relatively attractive, differentiation of (3.2) shows that demand increases for any given price p^* as μ rises. As diversity increases, customers become less sensitive to price (more inelastic demand) and these two effects imply prices necessarily rise. Likewise profits will increase. The effects on output are ambiguous. As shown in Anderson and de Palma (1985), when not buying is sufficiently attractive, outputs rise because the demand-evening effect of increasing μ dominates the elasticity-reducing effect.

4. Free Entry Equilibrium

The free entry equilibrium is characterized by zero profit for all firms and for the moment we treat the number of firms as a continuous variable. We denote free entry equilibrium values of variables by the superscript f.

Proposition 2: In a free entry equilibrium price, output per firm, and number of firms are given by:

$$p^f = c + \frac{K}{N} + \mu, \tag{4.1}$$

$$N\mathscr{P}^f = KN/(K + \mu N), \tag{4.2}$$

$$n^f = 1 + \frac{1}{\bar{K}} - \exp[1 + \chi + \bar{K}] \tag{4.3}$$

where $\bar{K} \equiv \dfrac{K}{N\mu}$ *and* $\chi \equiv [V_0 - a + c]/\mu$.

The Proposition is proved by setting profit (2.7) equal to zero and using (3.1) and (3.2) from Proposition 1. The comparative static results corresponding to Table 1 are displayed in Table 2.

TABLE 2
Comparative static results for the free entry equilibrium

	Output per Firm $N\mathscr{P}$	*Price* p	*Number of Firms* n^f
N	+	−	+
$V_0 - a$	0	0	−
c	0	+	−
K	+	+	−

The comparative static results for an increase in external competition, V_0, can be explained as follows. From Table 1, we know that, for a given number of firms, profits decrease when V_0 rises. Hence, the free-entry equilibrium number of firms is lower. From Table 1, it is also apparent that, *ceteris paribus*, price tends to increase as the number of firms decreases; however, more external competition tends to reduce price. These two effects exactly cancel in the present formulation.

We now consider a shift in the marginal cost, c. Output per firm, $N\mathscr{P}^f$ is unaffected; this implies that the change in price and number of firms exactly offset each other in equation (3.1). Equilibrium price increases by exactly the cost increase (see (4.1)); hence, in this case, cost increases are passed on 100% once we account for free entry of firms.

Finally, greater taste heterogeneity (measured by μ) decreases output per firm, $N\mathscr{P}^f$, but equilibrium price increases (see (4.1) and (4.2)). Since profit increases for $n^f \geqslant 2$ as μ increases (see Section 3), the number of firms increases.

The condition for the market to be served is $n^f \geqslant 1$. Using equation (4.3), the market is served if:

$$\chi \leqslant -1 - \bar{K} - \ln \bar{K} \tag{4.4}$$

It is easy to check that it is more likely that this will hold for large values of N and for small values of K and c. The effect of μ is ambiguous; as shown in Section 3, profit may increase or decrease with μ. Note that whatever the values of N, K, and μ there is always a level of external competition, V_0, such that the market will be served. For $\mu \to 0$, condition (4.4) reduces to

$$Nc + K \leqslant [a - V_0]N \tag{4.5}$$

In this case the market will be served by a single firm (Bertrand competition would drive price down to marginal cost if there were more firms), which will charge a price $p^m = a - V_0 + \delta$, where δ is an arbitrarily small constant. This price will enable the monopolist to serve the whole market so that the RHS of (4.5) is monopoly revenue and the LHS monopoly cost.

5. Social Optimum

In this section, we consider both first best and second best social optimum price, output and number of firms (range of product diversity). The welfare function, W, is the sum of consumer surplus and the profits of firms. The expression for consumer surplus has been derived by Small and Rosen (1981) for the logit model as (up to a constant):

$$CS = N\mu \ln \sum_{j=0}^{n} \exp[V_j/\mu]. \tag{5.1}$$

The profit function, π_i, is given by (2.7) with $\mathscr{P}_i$ defined by equation (2.4). Therefore the welfare function, W, is given by:

$$W = N\mu \ln \left[\sum_{j=0}^{n} \exp[V_j/\mu] \right] + N \sum_{j=1}^{n} (p_j - c) \frac{\exp[V_j/\mu]}{\sum_{i=0}^{n} \exp[V_i/\mu]} - nK. \tag{5.2}$$

Let the superscript w denote first best optimal values of variables. Noting that the first best optimum entails marginal cost pricing, the following result is readily shown by differentiating (5.2) and using (2.4):

Proposition 3: The prices, p^w, output per firm, $N\mathscr{P}^w$, and number of firms, n^w, which maximize the welfare function are given by:

$$p^w = c, \tag{5.3}$$

$$N\mathscr{P}^w = \frac{K}{\mu}, \tag{5.4}$$

$$n^w = \frac{1}{\bar{K}} - \exp \chi, \tag{5.5}$$

where $\bar{K}$ and χ are defined under Proposition 2.

The optimal number of firms, n^w, increases when N increases and decreases when K, V_0 and c increase. Note that for $n^w \geqslant 1$, $\partial n^w/\partial \mu > 0$: the more

consumers appreciate diversity, the larger the number of firms. Apart from this case, the intuition underlying the comparative static results is the same as the free entry case. For $n^w \leqslant 1$, the analysis below is relevant. Given that the welfare function is concave in n, the condition for the market to be served is now $W(n=0) \leqslant W(n=1)$. Using equations (5.2) and (5.3), after a few manipulations, this condition is:

$$\chi \leqslant -\ln[-1+\exp \bar{K}]. \tag{5.6}$$

Thus, it is more likely that the market be served for large values of N, V_0 and μ, and for small values of K and c. Except for μ, these results are identical to those determined in Section 4 for the condition under which the market will be served.[5] The optimal integer number of firms can be determined by a similar method.[6]

In the second best optimum, the sum of consumer and producer surplus is maximized subject to the constraint that firms earn non-negative profits. In the first best optimum firms are subsidized to the full amount of their fixed costs; now lump-sum transfers are either infeasible or prohibitively costly.

The welfare function is the same as above, but subject to non-negative profit constraints for all firms. The welfare problem is thus:

$$\max_{\{\mathbf{p};\, n\}} W = \mu N \ln\left(\sum_{j=0}^{n} \exp[V_j/\mu]\right) + N \sum_{j=1}^{n} \mathscr{P}_j(p_j - c) - nK,$$
$$\text{s.t. } N\mathscr{P}_j(p_j - c) = K \quad \text{for all } j = 1, \ldots, n. \tag{5.7}$$

Let a superscript s denote a second best optimal value of a variable.

Proposition 4: The prices, output per firm and number of firms which maximize the constrained welfare function (5.7) *are given by*:

$$p^s = c + \mu, \tag{5.8}$$

$$N\mathscr{P}^s = \frac{K}{\mu}, \tag{5.9}$$

[5] The proofs are trivial except perhaps for μ; in this case, as μ increases, Condition (5.6) for the market to be served becomes weaker:

$$\frac{\partial}{\partial \mu}\{-\mu \ln(-1+\exp \bar{K})\} = \frac{\bar{K}\exp \bar{K}}{-1+\exp \bar{K}} - \ln(-1+\exp \bar{K}) > 0$$

where $\bar{K} = \dfrac{K}{N\mu}$.

[6] If we let $\bar{n}^w = I(n^w)$, the number of firms maximizing welfare could be either $\bar{n}^w$ or $\bar{n}^w + 1$. The switch point from $\bar{n}^w$ to $\bar{n}^w + 1$ is given by the solution to $W(\bar{n}^w) = W(\bar{n}^w + 1)$. Hence $\bar{n}^w + 1$ firms will give rise to a larger level of welfare than $\bar{n}^w$ firms if

$$\chi < -\ln[-1+\exp \bar{K}] + \ln[1 + \bar{n}^w(1-\exp \bar{K})].$$

This formula generalizes formula (5.6).

$$n^s = \frac{1}{\bar{K}} - \exp[1 + \chi] \tag{5.10}$$

Proof: See Appendix 2.

From the reduced form expression for the second best number of firms, n^s, (equation (5.10)) we have the same qualitative comparative static effects for the second best situation as for the first best and the free entry cases. The condition for the market to be served is exactly the same as the free entry case: if the market is served, then, using the second best optimum as the welfare criterion, it should be served. To see why, note that the market is not served in equilibrium whenever it is not possible to make a profit. Such circumstances imply the zero profit constraint of the second best optimum cannot be satisfied. As soon as a firm finds it profitable to enter, then the second best optimum is also one firm as there is positive consumer surplus associated with introduction of the product and the constraint is met.

6. Equilibrium and Optimal Product Diversity

Spence (1976) identifies two opposing forces determining the degree of equilibrium product variety relative to the unconstrained social optimum. First, as firms are concerned with revenues rather than directly with consumer surplus and as revenues will not in general capture the entire consumer surplus associated with the introduction of a new product, there will be a tendency for too few firms in equilibrium. Second, a firm entering the market does not account for the loss in producer surplus (firm profits) inflicted on other firms. This is a tendency toward too many firms in the market. For the present model, we know from the discussion in 3 that total industry revenues net of variable production costs (TIR) are at first increasing and eventually decreasing as the number of firms in the industry rises. For example, when the number of firms is large, the product spectrum is well-covered and an extra firm would not be expected to add greatly to consumer surplus. *A priori*, we should therefore expect the first effect to dominate the second when n^f is small; this intuition is borne out in the analysis below.

In the first example discussed by Dixit and Stiglitz (1977) in their analysis of the CES representative consumer model, the solutions for the free entry equilibrium and the constrained social optimum coincide. Furthermore, they also have the same output for the unconstrained optimum as for these two. In the present notation, this implies $\mathscr{P}^f = \mathscr{P}^w = \mathscr{P}^s$. As evidenced by the reduced form equations for price and number of firms, these results do not hold in the present model (see Table 3 and the following discussion).

We find that output under the first and second best optima coincide (as do Dixit-Stiglitz), however, the free entry equilibrium involves lower output. The social optimum then tends to exploit economies of scale (due to the fixed cost) beyond the level of the free-entry equilibrium. Hence $p^f > p^s > p^w$. The greater

TABLE 3
Comparison between equilibrium and optimum
$\chi \equiv (V_0 - a + c)/\mu$; $\bar{K} \equiv K/N\mu$.

	Free Entry Equilibrium (f)	*First Best Welfare Optimum (w)*	*Second Best Optimum (s)*
Price	$c + \mu + K/N$	c	$c + \mu$
Output per firm	$KN/(K + \mu N)$	K/μ	K/μ
Number of firms	$1 + 1/\bar{K} - \exp[\chi + \bar{K} + 1]$	$1/\bar{K} - \exp\chi$	$1/\bar{K} - \exp[\chi + 1]$

is the preference for diversity, μ, the greater the second best price—a larger range of products must be financed, given the non-negative profit constraint for firms, by higher prices. This is also true of the market solution, although in that context a larger μ imparts more monopoly power. To shed further light on these results, we now consider the different solutions for optimal product diversity.

It is clear from Table 3 that $n^s < n^w$. Note also that n^f, n^w and n^s are all decreasing and concave functions of χ. These schedules are illustrated in Figure 1.

Note that $\mathrm{d}(n^w - n^w)/\mathrm{d}\chi > 0$; for the limit case $\chi \to -\infty$, both n^w and n^s tend to $1/\bar{K}$ whereas n^f tends to one firm greater.

We now compare the free-entry equilibrium to the first-best optimum number of firms. The next result is proved from Table 3.

Proposition 5:

For $n^w = 0$, $n^f = 0$,
For $n^w > 0$, *either* $n^w > n^f$ *or* $1 > n^f - n^w > 0$, *with* $\mathrm{d}(n^f - n^w)/\mathrm{d}\chi < 0$
and $\lim_{\chi \to -\infty} (n^f - n^w) = 1$.

This shows that the market is not served when it should not be served but it may not be served when it should be. Indeed it may be the case that it is not served when the first best optimum would require a large number of firms. However, when there is over-entry, it is by at most one firm.

If the market solution yields 'too many' firms (which occurs for χ relatively small), then the difference between equilibrium and first best optimal product diversity is negligible.[7] This suggests that some care should be taken in

[7] As pointed out by a referee, Mankiw and Whinston (1986) also find a one firm result in comparing first best optimum to equilibrium—except that the result goes the opposite way, i.e. $n^f \geqslant n^w - 1$. Their analysis concerns homogeneous products—for our model with $\mu \to 0$, we have $n^w = n^f = 1$ for $K \leqslant N(a - V_0 - c)$ and $n^w = n^f = 0$ otherwise.

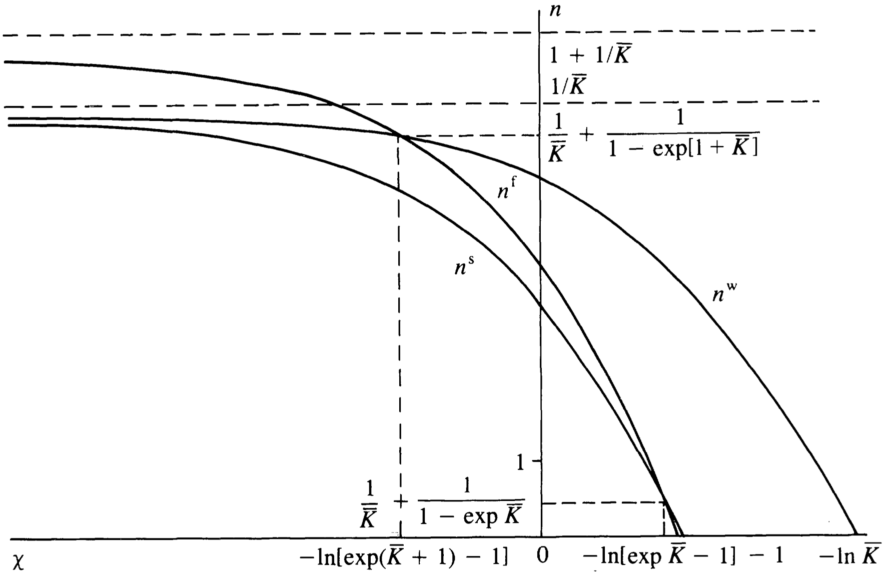

FIG. 1. *Equilibrium and optimal numbers of firms*

interpreting previous results in the literature which consider only the sign of $(n^f - n^w)$ without looking at the magnitude of the difference. On the hand, when the market solution yields 'too few' firms, then the divergence between n^w and n^f may be severe. Recall that χ is a measure of the relative quality of the outside good. When the outside good is relatively unattractive (alternatively, the market provides relatively high qualities), the situation is a 'sort of ideal' even in a first best sense; when the market provides relatively low qualities it may provide way too few varieties.

The case $K \to 0$ (or equivalently, $N \to \infty$) can be interpreted as Chamberlinian monopolistic competition. From Table 3 we have $\lim_{K \to 0} (n^w - n^f) = (e - 1)\exp \chi - 1$, therefore:

$$n^w > n^f \quad \text{for} \quad \chi > \hat{\chi};$$

and
$$1 > n^f - n^w > 0 \quad \text{for} \quad \chi < \hat{\chi};$$

where $\hat{\chi} \equiv -\ln(e - 1)$. Qualitatively, the results are the same as for $K > 0$.

We now compare n^f and n^s. From Table 3, $0 < n^f - n^s < 1$ for $n^f \geqslant 1$. Hence n^f always exceeds n^s in the relevant range but the extent of over-entry when compared to the second best welfare benchmark is at most one firm. We therefore explicitly consider the integer number of firms to see whether both over- and under-entry are still possible. Define n_I^s as the second best optimal integer number of firms; n_I^f and n_I^w are analogously defined. Of course,

$n_I^f = I(n^f)$, where $I(n^f)$ is the integer part of n^f. By concavity of the welfare function, to compute n_I^s it is sufficient to compare $W^s(I(n^s))$ to $W^s(I(n^s)+1)$; and similarly for n_I^w.

Proposition 6

For $$n_I^f = 0,\ n_I^f = 0 \Leftrightarrow n_I^s = 0, \tag{6.1}$$

For $$n_I^f = 1,\ n_I^s = 1 \text{ or } 2, \tag{6.2}$$

For $$n_I^f > 1,\ n_I^s = n_I^f - 1,\ n_I^f \quad \textit{or} \quad n_I^f + 1. \tag{6.3}$$

In this Proposition, (6.1) is a direct consequence of the equivalence between the condition that the market will be served and that it should be served under the second best criterion (see Section 5); (6.2) and (6.3) follow from the fact that for $n^f > 1$, $0 < n^f - n^s < 1$ and from the fact that $n_I^f = I(n^f)$ and $n_I^s = I(n_I^s)$ or $I(n_I^s) + 1$ (by concavity of W). The example illustrated in Figure 2 demonstrates that all the possibilities of (6.3) may occur.

An interesting situation occurs for, say, $\chi = 0.5$, where no firm will enter the market although it would be socially optimal (first best) to have three firms in the market. Note secondly that it is not always the case that the free entry and unconstrained optimal numbers of firms will coincide for sufficiently small V_0 once we account for the integer problem. For example, if $\bar{K} = 0.196$, the free entry equilibrium number of firms is six for $\chi < 3.5$. However, the first best number cannot exceed five regardless of χ.

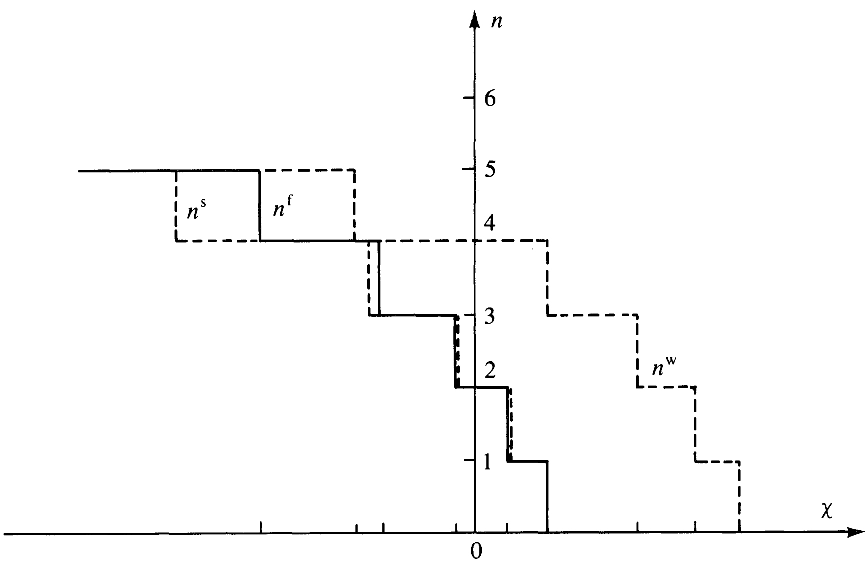

FIG. 2. *Equilibrium and optimal integer numbers of firms*: $\bar{K} = 0.2$

7. Conclusions

This paper has introduced the logit model of discrete choice as an analytical tool into standard oligopoly analysis and used it to address the question of market and optimal product diversity. As shown elsewhere (Anderson et al. (1988) and (1989)), the logit can be derived from preference foundations which are familiar to economists in industrial organization and it provides a convenient representation of the degree of heterogeneity of consumer tastes. As an aggregate functional form to study demand for differentiated products produced by oligopolists, it was shown to have properties which are reasonable in this context. The model also gives rich comparative static results—in particular with respect to taste heterogeneity (μ) and the attractiveness of non-purchase (V_0)—which can be interpreted intuitively.

The present model involves lower output per firm then either the first or second best, so the equilibrium takes less advantage of economies of scale. As the market price exceeds the price in the unconstrained optimum, this *per se* does not imply there are too many firms. Indeed, when there are very few firms in the market equilibrium, it is likely that there are *too* few. However, as the number of firms in equilibrium increases (a situation which can be construed as monopolistic competition) then the equilibrium may exceed the optimum (although the two are very close). This result should be contrasted with the conclusions of Salop (1979) who found, for a model where firms are located around a circle, that competition will always yield too much variety (too many firms). This difference is probably due to the fact that in our model all products compete directly with all others so competition is nonlocalized and over-entry is less likely.

The logit approach is flexible enough to accommodate consideration of a wide range of strategic variables available to firms (such as price, geographical location, advertising, product quality, number of products to offer etc.). We feel that there is a broad range of potential theoretical applications for the logit formulation which could be supported by empirical studies since the logit is already well-established in econometrics.

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APPENDIX 1

Proof of Proposition 1.

The first-order conditions corresponding to (2.7) are given by

$$p_i = c + \frac{\mu}{1 - \mathscr{P}_i}, \quad i = 1 \ldots n \tag{A.1}$$

We first prove that the only candidate equilibrium is the symmetric one given by (3.1) and (3.2). Note that these equations have a unique solution, p^*, so that only one symmetric equilibrium exists. The proof proceeds by contradiction. Suppose there is another equilibrium which is not symmetric. Denote by $p^+ > p^*$ the highest price in this equilibrium with $\mathscr{P}^+$ the corresponding probability. If all prices had risen to p^+, $\mathscr{P}^+$ would be less than $\mathscr{P}^*$. *A fortiori*, given some other prices are smaller than p^+, $\mathscr{P}^+$ is necessarily smaller than $\mathscr{P}^*$. Given that p^* and $\mathscr{P}^*$ solve (A.1), then it cannot be solved by a higher price and a lower value of $\mathscr{P}_i$ (the RHS is increasing in $\mathscr{P}_i$). A similar argument shows that it is not possible for any price to be less than p^*.

It remains to be shown that profits are quasi-concave in own price so that (A.1) does indeed constitute the firm's reaction function. First. (A.1) has a unique solution, p_i, for any vector of other firms' prices, p_{-i}. Second, the second derivative of the profit function, evaluated at (A.1), is $-\mathscr{P}_i(\mathscr{P}_i - 1)^2/\mu$ which is therefore negative at any turning point so profit is quasi-concave in p_i.

APPENDIX 2

The second best problem

For the second best optimization problem, the zero profit condition is written as $(p_i - c)N\mathscr{P}_i = K$, or:

$$\frac{N}{K}(p_i - c)\exp[(a - p_i)/\mu] = \sum_{j=1}^{n^s} \exp[(a - p_j)/\mu] + \exp[V_0/\mu]. \tag{A.2}$$

Substituting into the welfare function (5.2) yields

$$W^s = N\mu \ln\left\{\frac{N}{K}(p_i - c)\exp[(a - p_i)/\mu]\right\}.$$

Therefore, at the optimum, all firms charge the same price given by $p^s = c + \mu$. The second order condition is readily verified at this solution. From (A.2) we then have

$$n^s = N\mu/K - \exp[(\mu + V_0 - a + c)/\mu].$$

Again from (A.2), $N\mathscr{P}^s = K/\mu$.